Abstract Algebra Tutorial Week 2 Handout

02/10/17

1 Introduction

Tutor Name: Alexandre (Alex) Daoud (S5.13)
Tutorial Group: 02
Tutorial Website: `http://www.p-adic.com/teaching/ccm121_1819`

My notes on *Basic Set Theory* may be helpful for this course - they can be found on my site `http://www.p-adic.com`.

2 Set Theory

2.1 Sets

The convention on whether or not $\mathbb{N}$ contains 0 is not widely agreed upon. Most pure mathematicians agree that $\mathbb{N}$ contains 0 since that is what most constructions of $\mathbb{N}$ yield. However this course assumes that it does not contain 0 (perhaps for simplicity) so we shall ignore the existence of other conventions.

The set declaration notation in the following example

$$A = \{\, n \in \mathbb{N} \mid n \geq 0 \,\}$$

is often referred to as **set-builder notation** with the statement after the '|' or ':' being called the **entrance requirement** or **entry condition**.

Note that not every 'collection' of 'things' can honestly be called a 'set'. This might sound like utter nonsense to you but consider the following 'example'. Let X be the 'set' of all sets that do not contain themselves. Can X honestly be a set? Not really since we have that

$$X \in X \iff X \notin X$$

which is clearly a contradiction - it states that X is contained in X if and only if X is not contained in X (try proving this yourself). This is better known as Russel's paradox and formed part of the motivation for mathematicians to come up with a consistent theory of sets. In such a theory, the above 'collection' is disallowed from being a set.

2.2 Intervals

For those of you coming from the French system (or have experience in it), you may recognise $]a, b[$ for open intervals. This convention is not used very much, if at all, in the English system where (a, b) is used.