Numbers and Functions Tutorial Week 12 Handout

13/11/17

1 Comments on Tutorial Answers

1.1 General comments

- Don't use clsoed brackets on ∞! Closed brackets means the pbject in the interval is intended to be an element of the set. But ∞ isn't a number so it is always written with a round bracket:

$$(-\infty, -1) \textbf{ IS CORRECT, NOT } [-\infty, -1)$$

1.2 Question 1

First of all, there is a typo in the solutions on KEATS. The negation should read:

$$\exists \delta > 0 \text{ such that } \forall m \in \mathbb{N} \quad \exists n \geq m \text{ such that } -\delta \geq \frac{(-1)^n}{n} \text{ or } \frac{(-1)^n}{n} \geq \delta$$

I'm still seeing some people mixing up when they should negate inequalities. The golden rule is as follows.

If an inequality comes immediately after (attached to) a quantifier ($\forall$ or $\exists$), it stays the same. If it does not then it gets negated.

So in the example above, the negation of $\forall \delta > 0$ is $\exists \delta > 0$; however the negation of $-\delta < \frac{(-1)^n}{n}$ is $-\delta \geq \frac{(-1)^n}{n}$.

Another point I want to make is to make sure you change the English around when negating so that your negation makes sense grammatically. This isn't a mathematical thing but rather a point about writing mathematical prose. When someone is reading your math, it makes it much easier for them to decipher the meaning when the English flows nicely.

1.3 Question 2

Some people had forgotten the closed brackets on the ends of the intervals. For example in 2.a), $S_- = (-\infty, 1]$ since 1 is a lower bound for this set.

1.4 Question 3

There was some confusion as to what exactly we are doing when we are proving that for all $\varepsilon > 0$ there exists $x \in S$ such that $x < m + \varepsilon$ where $m = \inf S$. Let me give you an intuitive viewpoint for this (for sup just do the mirror image of the argument).

Fix a $\varepsilon > 0$. Let's call ε a **pertubation**. If $m = \inf S$ then the statement

$$\exists x \in S \text{ such that } x < m + \varepsilon$$

is saying that, no matter how small you perturb m by with your pertubation ε, you are always guaranteed that $m + \varepsilon$ will land in S (or will land in the set of uppwer bounds of S). So you will always be able to pick an $x \in S$ so that you can squeeze x between m and $m + \varepsilon$. Since I don't have much time to typset a picture for you, I will give you the picture in class and you can fill it in below:

(fill in here)