Numbers and Functions Tutorial Week 11 (apparently...I thought it was Week 6) Handout

06/11/17

1 Comments on Tutorial Answers

1.1 General Comments

- I am still seeing answers that look like this:

  $$x < -7 \cup x > 3$$

  Remember - $\cup$ and $\cap$ are operations on **sets**. $x < -7$ and $x > 3$ are not sets, they are **statements**. They can be made into sets as follows:

  $$\begin{aligned} x < -7 &\implies \{\, x \in \mathbb{R} \mid -\infty < x < -7 \,\} \\ x > 3 &\implies \{\, x \in \mathbb{R} \mid 3 > x > \infty \,\} \end{aligned}$$

- When using set-builder notation, make sure to write which set the elements you are considering originate from. For example, write

  $$\{\, x \in \mathbb{R} \mid x > 0 \,\}$$

  instead of

  $$\{\, x \mid x > 0 \,\}$$

- Remember, round brackets are for intervals of real numbers:

  $$(2, 4) = \{\, x \in \mathbb{R} \mid 2 < x < 4 \,\}$$

  and curly brackets are to denote sets:

  $$\{\, 2, 4 \,\} = \text{ the set containing only the elements 2 and 4}$$

1.2 Question 1

Note that at the beginning of this question, it is stated that we are assuming $n \in \mathbb{R}$ is greater than 0.

c) e) f) A common slip-up in these questions was to use $\log_{1/2}(x)$ but not take into consideration how a base smaller than 1 affects the properties of the logarithm. In fact, it is common to take the definition

$$\log_{1/n}(x) = -\log_n(x)$$

Try plotting $\log_{1/2}(x)$ and $\log_2(x)$ with WolframAlpha to see the difference graphically.

1.3 Question 2

When dealing with equations with an absolute value (a.k.a modulus) you should consider the cases where the contents of the absolute value are both positive and negative. For example

$$\begin{aligned} |x+2| > 5 &\iff (x+2 > 5) \text{ or } (-(x+2) > 5) \\ &\iff (x+2 > 5) \text{ or } (-(x+2) > 5) \\ &\iff (x > 3) \text{ or } (-x-2 > 5) \\ &\iff (x > 3) \text{ or } (-x > 7) \\ &\iff (x > 3) \text{ or } (x < -7) \\ &\iff x \in (3, \infty) \cup (-\infty, -7) \end{aligned}$$

2 Question 3

e) A common mistake here was to write $S_+ = [1/2, \infty)$. Note that $1/2$ is not an upper bound for S. Indeed, given large enough even n we have

$$(-1)^n - \frac{1}{n} = 1 - \frac{1}{n} > \frac{1}{2}$$

In fact, any even $n > 2$ works. This set has no maximum but its supremum is indeed 1. Hence $S_+ = [1, \infty)$.