Numbers and Functions Tutorial Week 5 Handout

23/10/17

1 Comments on Tutorial Answers

1.1 General

- When negating propositions, we don't negate the inequalities that are attached to a quantifier. Indeed, consider the statement

$$\forall \underbrace{\varepsilon > 0}_{\textbf{not negated}} \, \exists N \in \mathbb{N} \text{ such that } \forall \underbrace{n \geq N}_{\textbf{notnegated}} \text{ we have } \underbrace{|s_n - l| < \varepsilon}_{\textbf{negated}}$$

1.2 Question 1

a) b) For these questions, I would be quite explicit. Instead of

Not all students in the class that is not right handed

I would write

There exists a student in the class who is left handed (or ambidextrous)

Also for this question - the negation of 'there is' is 'for all' **not** 'there is not'. Translate the words to symbols if you are confused what the negation of a logical statement is. It is not always the case that the negation of a logical statement will coincide with its 'negation' in English.

1.3 Question 7

d) Let $A = \{\, x \in \mathbb{R} \mid x^2 \geq 10 \text{ AND } 2x + 1 < 0 \,\}$. Then

$$\begin{aligned} x \in A &\iff ((x \leq -\sqrt{10}) \text{ OR } (x \geq \sqrt{10})) \text{ AND } x < -\frac{1}{2} \\ &\iff x \in (-\infty, -\sqrt{10}) \end{aligned}$$

1.4 Question 8

Here is a proof for this question that doesn't involve induction.

Proposition. *Let A be a set of finite cardinality n. Then there are $n!$ distinct bijections $f : A \to A$.*

Proof. We shall construct $n!$ bijections $f : A \to A$ as follows. Choose $x \in A$ and consider $f(x)$. There are n choices for $f(x)$. Say $f(x) = a$ for some $a \in A$. Now let $y \in A$ be different from x. There are $n - 1$ choices for $f(y)$. Indeed, we cannot take $f(x) = a$ since otherwise f would not be injective. Hence choose $b \in A$ different from a and set $f(y) = b$. We may continue in this fashion until we exhaust all elements of A. It is clear that at each stage we have one less choice for the image of our element of A. This constructs a single bijection $f : A \to A$. Since, at each stage, we were free to choose any value up to the allowed amount, we get $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$ bijections. Furthermore, it is clear that there can be no more bijections and we have exhausted all elements of A. $\square$

2 Comments on Lecture Notes

2.1 Boundedness; supremum and infimum

You can often think of the supremum and infimum as what the maximum and minimum should be were they to exist. This is quite similar to the idea of a limit being the definition of a function/sequence at a point were it to be defined at that (likely undefinable) point.